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# The zone plate as a television test pattern

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Research Department, Engineering Division
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## Summary

*The zone plate pattern is a two-dimensional linear sweep of spatial frequency which can be used to give a quick appraisal of a television system's frequency characteristics. Applications discussed include measurement of filter characteristics, source and display properties, sub-Nyquist sampling system impairments, PAL coding and decoding and standards conversion. The pattern can be generated electronically in both circular and hyperbolic form, although the latter is simpler and the circuit diagram of such a generator is provided.*

Issued under the authority of

Head of Research Department

**Research Department, Engineering Division,**
**BRITISH BROADCASTING CORPORATION**
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# THE ZONE PLATE AS A TELEVISION TEST PATTERN

J.O. Drewery, M.A., Ph.D., C.Eng., M.I.E.E.

## 1. Introduction

The zone plate pattern is now being used extensively as a diagnostic tool for the analysis of television systems. The purpose of this report is to explain its properties and give examples of how it can be used.

The circular form of zone plate shown in Fig. 1 is probably the one that students of optics are most familiar with.[1] However, the zone plate can take other forms where the lines of constant phase are general conic sections. In particular the rectangular hyperbolic form shown in Fig. 2 has been found to be most useful. Most of the discussion which follows will be in terms of the circular zone plate, but the arguments are easily adapted to the hyperbolic zone plate by exchanging horizontal and vertical properties.

The important property of the zone plate pattern, as far as television is concerned, is that it is a two-dimensional linear sweep of spatial frequency. Fig. 3 shows the lines of constant phase for a general spatial wave, representing a small area of the pattern, of wavelength $\lambda$ which has resolved wavelengths along the horizontal and vertical axes of $\lambda_x$ and $\lambda_y$. Spatial frequency is the reciprocal of wavelength and thus the wave can be resolved into horizontal and vertical frequency components $f_x$ and $f_y$ which are the reciprocals of $\lambda_x$ and $\lambda_y$. Wave fronts parallel to the $x$ axis have $f_x$ equal to zero; wave fronts parallel to the $y$ axis have $f_y$ equal to zero.

In the zone plate patterns the spatial frequency clearly increases towards the edges, and it is shown in Appendix 1 that the circular zone plate has the property that $f_x$ is proportional to $x$ and $f_y$ is proportional to $y$ where $x$ and $y$ are measured from the centre of the pattern. Thus horizontal frequency is proportional to horizontal distance and vertical frequency is proportional to vertical distance and each point on the zone plate is associated with a local spatial frequency whose $x$ and $y$ components are proportional to their respective distances from the origin.

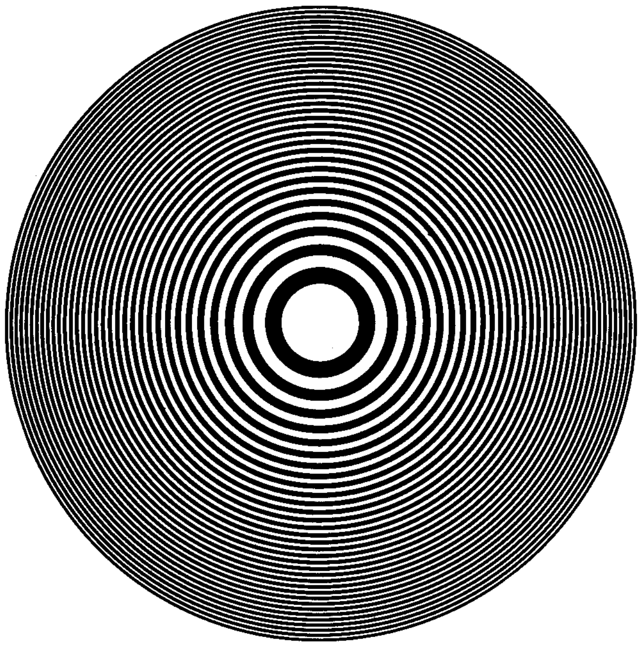

*Fig. 1 - A circular zone plate*

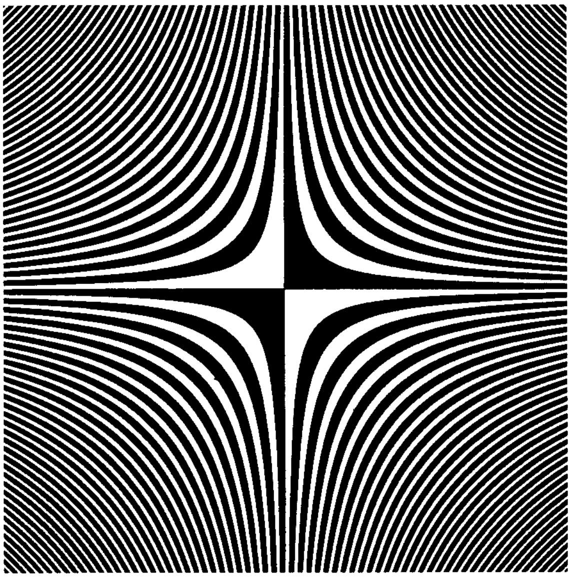

*Fig. 2 - A rectangular hyperbolic zone plate*

For the rectangular hyperbolic zone plate $f_x$ is proportional to $y$ and $f_y$ is proportional to $x$ so that horizontal frequency increases vertically and vertical frequency increases horizontally. Thus the rectangular hyperbolic zone plate is like the circular zone plate but turned inside-out.

With a circular zone plate it is self-evident that the magnitude of the spatial frequency is constant at points on a circle centred on the origin. However, this is also true for the hyperbolic zone plate as can be deduced by taking the square root of the sum of squares of the spatial frequency components.

## 2. Applications

### 2.1. Filter characteristics

As the zone plate is a linear sweep it can be used to obtain a quick appraisal of the frequency characteristic of any intra-field filter operating on the video signal. This can be done by scanning a zone plate and observing a picture monitor. Such a method gives only a rough outline of the characteristic but for more quantitative information the video signal can be observed on an oscilloscope.

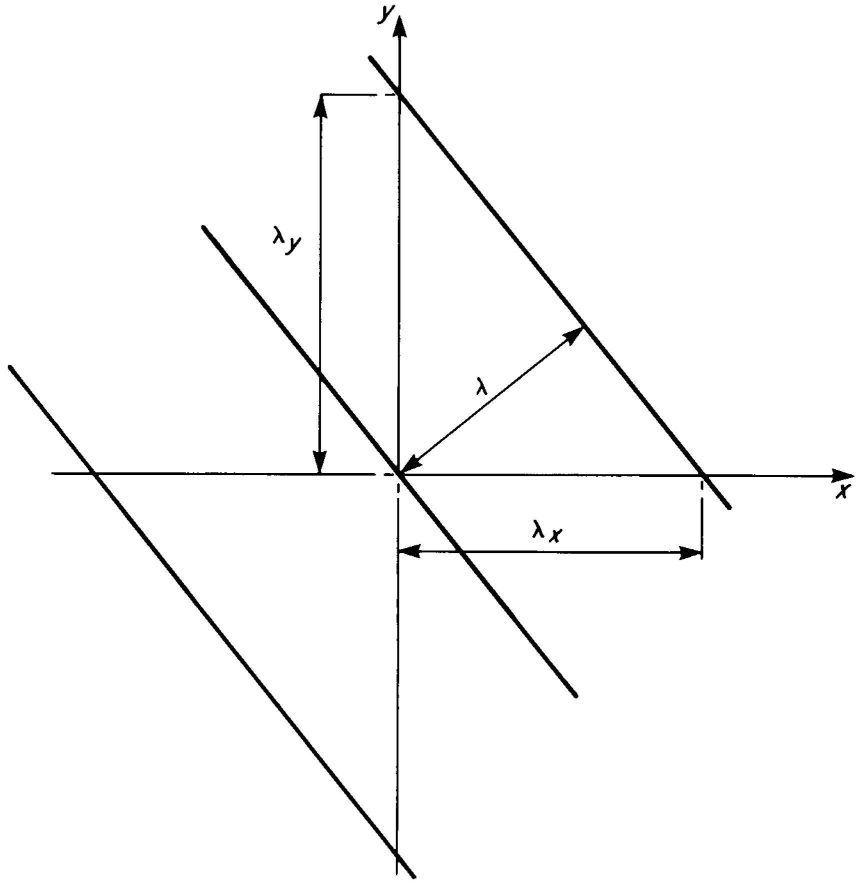


*Fig. 3 - A general spatial wave showing resolution into component wavelengths*

For example a conventional low-pass filter limits the horizontal frequency component independently of the vertical frequency component. Thus the effect of the filter is to pass all frequencies between the parallel vertical lines as shown in Fig. 4(*a*). Beyond the pass region the display changes to a uniform grey.

A vertical low-pass filter, such as could be obtained by combining the signals of adjacent lines, limits the vertical frequency independently of horizontal frequency. Thus the pass region lies between the two horizontal parallel lines as shown in Fig. 4(*b*).

**2.2. Source and display characteristics**

It can be shown[2] that the action of scanning a scene with an arbitrary aperture has the effect of prefiltering the scene with a filter before scanning. The frequency characteristic of this filter is the Fourier transform of the aperture and this will be impressed on the zone plate when displayed. Thus a direct two-dimensional transform of the aperture is obtained.

For more quantitative measurement the oscilloscope waveform can be observed. Any point at a constant time

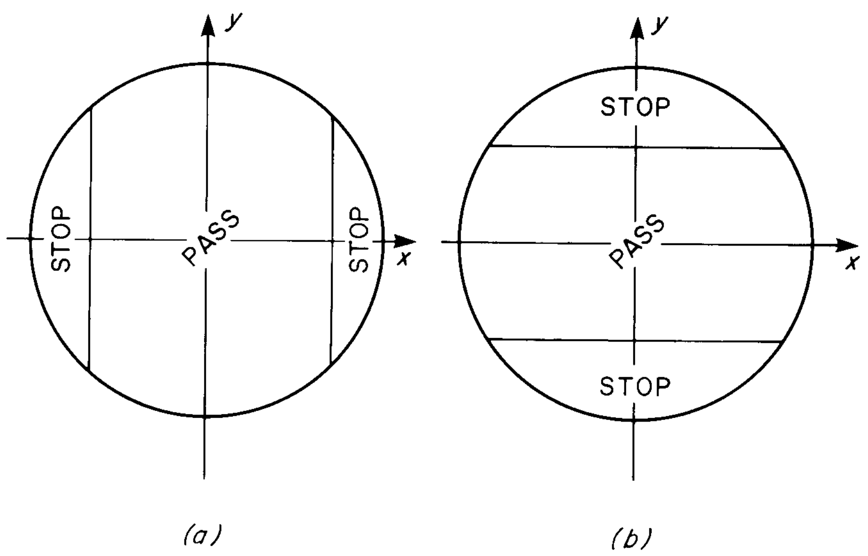


*Fig. 4 - The effect of low-pass filters on the zone plate pattern*
(*a*) horizontal (*b*) vertical

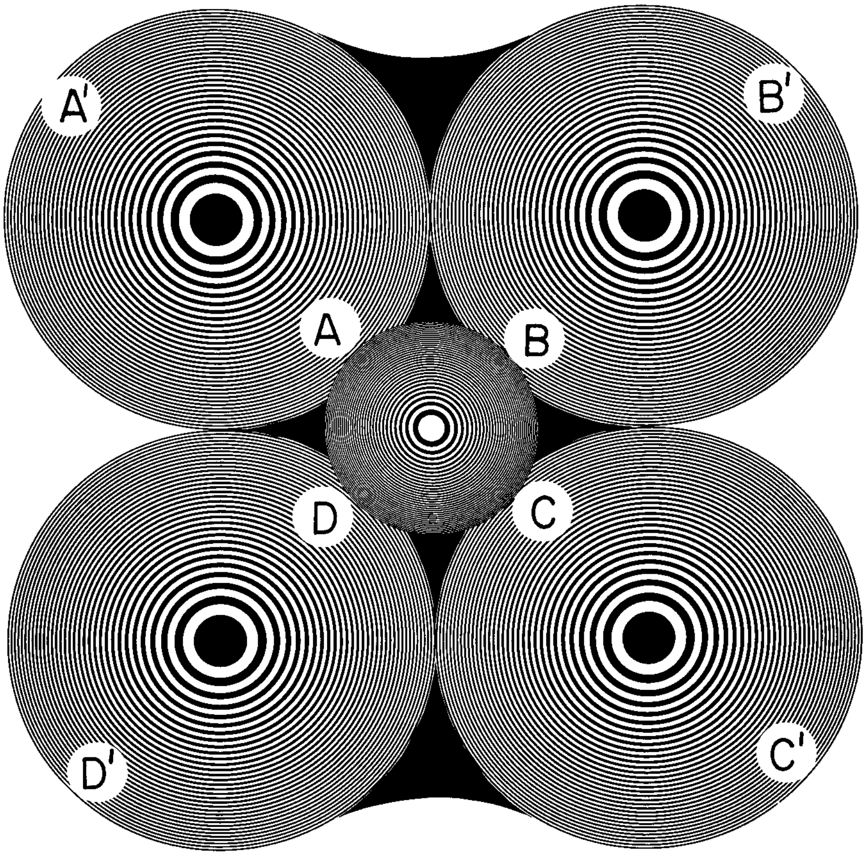


*Fig. 5 - The usual arrangement of circular zone plates*

after a line synchronising pulse represents a constant horizontal frequency component. Thus if the oscilloscope is triggered by line pulses the waveform is a solid profile of the aperture's horizontal Fourier transform (assuming this is independent of the vertical transform). In the same way, by triggering the oscilloscope with field pulses the profile of the aperture's vertical Fourier transform can be obtained.

There are two important points, however, about this method of measurement. First, it must be carried out on a signal which is linearly related to scene brightness because source aperture distortion arises in the linear domain. Second, a correction factor of $4/\pi$ must be applied to the region near the origin if the zone plate is of square-wave form.

When the zone plate is displayed, the final result is also a function of the display aperture.[2] Thus the display aperture's Fourier transform is also impressed on the zone plate and it is, in general, difficult to separate the effects of source and display. In extreme cases the strength of alias components caused by the scanning will indicate the dominant transform as will be shown later.

The effect of the source can be eliminated by using an electronically-generated signal having prescribed source properties. With electronic-generation it is also easy to make a sine-wave zone plate which does not need the $4/\pi$ correction factor. The manufacture of test charts and slides having sinusoidal intensity variation is well known to be very difficult.

With both sources and displays the aperture, and therefore resolution, varies over the picture area. It is therefore customary to have not one, but five zone plates arranged in a square as shown in Fig. 5. In the absence of variation, the amplitudes of the spatial effects seen at A′ would be identical to those at A, and so on, because the two-dimensional transform of a real function has a centre of symmetry for amplitude (and a centre of anti-symmetry for phase). With aperture variations, however, this symmetry is broken and must always be borne in mind when interpreting the display. Moreover, any disparity in the effects at A and C, B and D, etc. indicates an asymmetry in the spatial variation.

#### 2.3. Sampling structures

When a function is sampled its spectrum is repeated at multiples of the sampling frequency. Conventional horizontal scanning is a sampling operation having a sampling frequency of $N$ cycles/picture height where $N$ is the number of scanning lines per field. Thus an arbitrary spatial frequency having components $m$ cycles/picture width and $n$ cycles/picture height is accompanied, when scanned and displayed, by components $(m, n \pm N)$, $(m, n \pm 2N)$, etc. In particular the frequencies $(0, \pm N)$, $(0, \pm 2N)$ etc. are accompanied by zero spatial frequency.

Now this applies equally to the zone plate which contains all spatial frequencies. Upon each input frequency is impressed an infinite series of alias components. As the zone plate is a linear sweep it follows that the patterns of alias components are also linear sweeps, i.e. they appear as

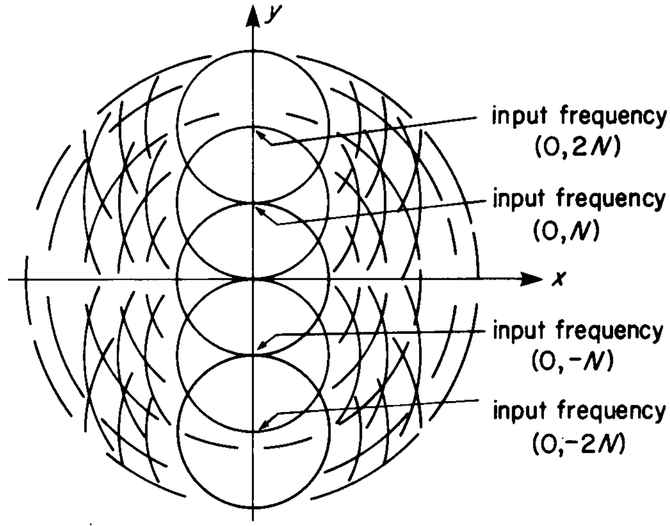


*Fig. 6 - Higher order alias components generated by vertical sampling*

further zone plates centred on the frequencies (0, ± $N$), (0, ± 2$N$) etc. as shown in Fig. 6. Normally the first members of this series are clearly visible and sometimes the second members are also visible.

With interlaced scanning, the scanning of the interlaced field inverts the odd spectra, i.e. those centred on (0, ± $N$), (0, ± 3$N$) etc. so that these spectra appear to flicker at the picture frequency. However, the even spectra appear steady. The visibility of the odd spectra depends on the temporal response partly of the display and partly of the eye. If a photograph were taken having an exposure time of one picture period the odd spectra would not be recorded (neglecting gamma effects) and the spatial scanning frequency would appear to be doubled.

The visibility of the higher-order spectra depends on the spatial apertures of the source and display as well as the temporal response of the display. If the source is 'soft' then the low-frequency alias components associated with high input frequencies will be feint. If the display is 'soft' then the high-frequency alias components associated with low input frequencies will be feint. Considering just the first-order spectra it is convenient to adjust the size of the main zone plates so that the centre of the first-order spectra lie on the edge of the figure as in Fig. 7. Then lack of alias components in region A indicates a 'soft' source

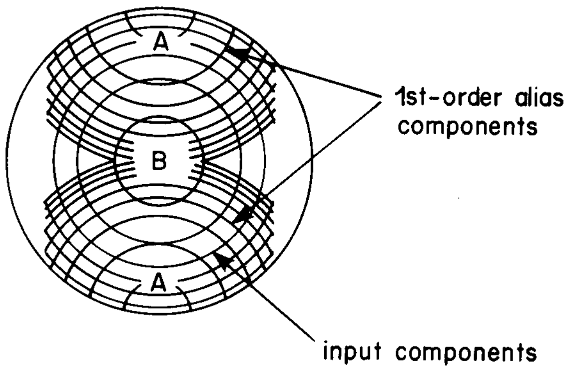


*Fig. 7 - The preferable adjustment of the zone plate pattern size relative to the vertical scanning pitch*

and lack at B indicates a 'soft' display. The strength of alias components usually rises to a maximum somewhere in between, if tending towards A indicating a 'soft' display and if tending towards B indicating a 'soft' source.

If the video signal is, itself, sampled this creates a further repetition of spectra where the further spectral

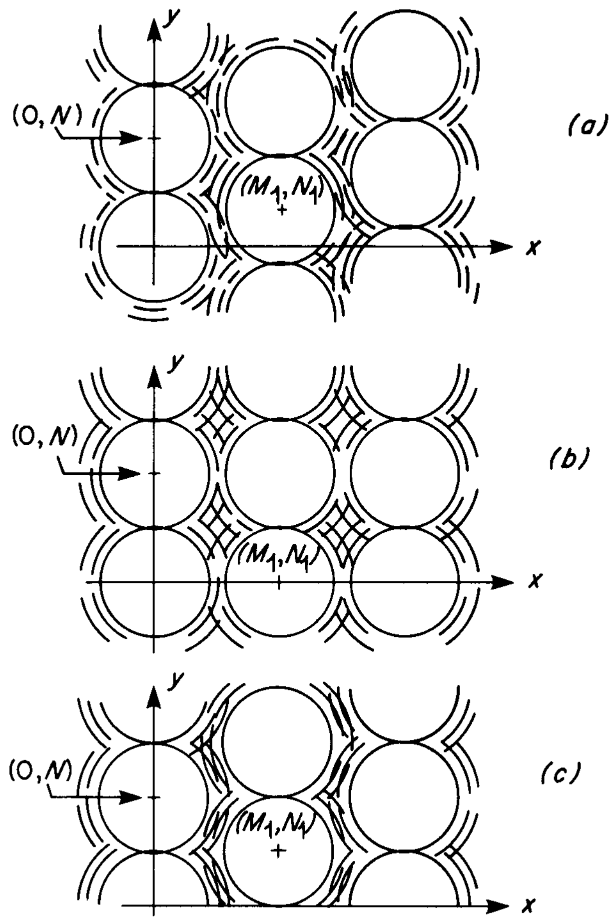


*Fig. 8 - Higher order two-dimensional alias components generated by sampling the video signal*
($a$) arbitrary sampling ($b$) line-locked sampling
($c$) half line-offset sampling

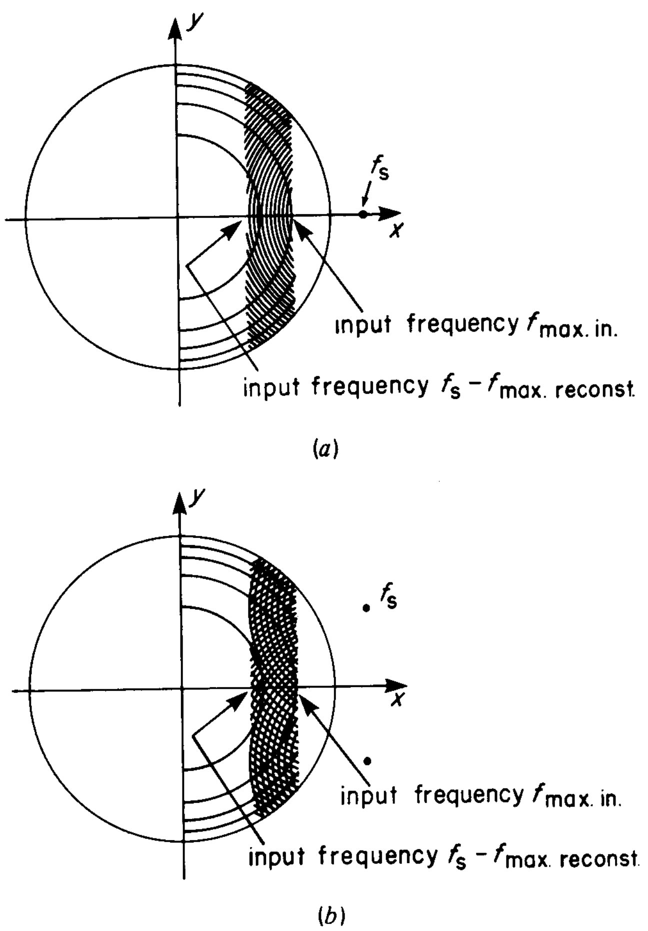


*Fig. 9 - The vestigial alias components generated by sub-Nyquist sampling of the video signal*
(*a*) line-locked sampling (*b*) half-line-offset sampling

repeat unit has the components of the video sampling frequency. If these components are $(M_1, N_1)$ and the samples are not interpolated before display then the sampling appears as further zone plates centred on multiples of $(M_1, N_1)$. These components are then repeated with the unit $(0, N)$, because of the normal scanning, to yield a two-dimensional array of zone plates as in Fig. 8(*a*). For example a line-locked sampling frequency would give an array as shown in Fig. 8(*b*); a half-line offset in the sampling frequency would give a pattern as in Fig. 8(*c*).

Fig. 8 is drawn assuming that the size of the zone plate is much larger than the spectral repeat units. In practice when the size of the pattern is adjusted relative to the scanning pitch as in Fig. 7 all these higher-order spectra are not seen. In fact if the signal has a well-defined bandwidth nothing is seen beyond the vertical lines representing the signal cut-off frequency. But the vestiges of the higher-order spectra caused by the sampling of the video signal remain in the central area and are only filtered out by the reconstruction filter and display. If the sampling is super-Nyquist then the vestiges fall outside the passband of the reconstruction filter and they are not therefore displayed. But if the sampling is sub-Nyquist[3] and the reconstruction filter is a simple low-pass type, part of the vestiges will lie in the passband and so will be displayed as in Fig. 9.

In Fig. 9(*a*) the sampling frequency, $f_s$, is line-locked and therefore has no vertical frequency component, in Fig. 9(*b*) the sampling frequency has a half-line frequency offset and so the alias components, centred on this offset, appear as diagonal patterns when the input is line-locked. This makes them less visible.

It is important to realise that the alias components appear in the positions occupied by the input frequencies that cause them. Thus in Fig. 9(*b*) if a post-comb-filter is used to remove the diagonal alias components it also removes the wanted diagonal input components. But it leaves the vertical alias components occupying positions where there is apparently no input as in Fig. 10.

### 2.4. PAL coding and decoding

If a monochrome zone-plate is decoded by a normal PAL decoder the presence of the high-frequency luminance causes cross-colour. This occurs because the chrominance circuits cannot distinguish between true chrominance and luminance near subcarrier frequency. Thus the luminance is demodulated and, as demodulation is equivalent to a

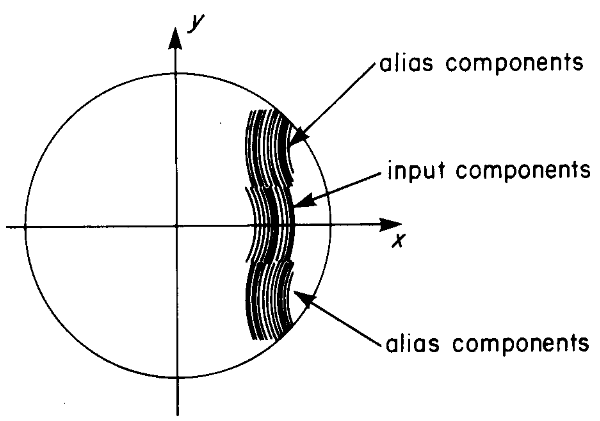


*Fig. 10 - Residual alias components generated by sub-Nyquist sampling of the video signal at half-line offset followed by comb-filtering*

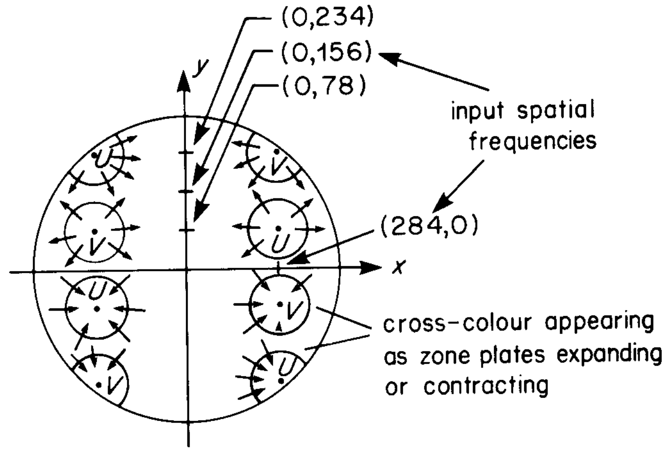


*Fig. 11 - The appearance of a PAL coded and decoded monochrome zone plate pattern. (Only the outline of the cross-colour components is shown)*

frequency shift, the cross-colour appears as further zone plates of the appropriate demodulator colour and centred on the appropriate carrier positions. These positions have spatial and temporal frequency co-ordinates of principally ±(284, 78, 18¾) and ±(283, −235, −6¼) for the $U$ chrominance and ±(284, 234, 6¼) and ±(283, −79, −18¾) for the $V$ chrominance.[4] Thus the cross-colour zone plates appear as in Fig. 11.

The third co-ordinate is the temporal frequency and occurs because the chrominance carrier frequencies are not picture-locked. They thus correspond to moving luminance and, in turn, cause stationary luminance to be decoded as moving cross-colour. The higher temporal frequency is associated with the lower vertical frequency so that the inner zone plates in Fig. 11 move faster than the outer ones. As the strength of the cross-colour is proportional to the luminance which causes it which, in turn, decreases statistically with increasing spatial frequency, the slower-moving cross-colour is at a lower level than the faster moving type. Because the phase effects seen on the zone plate have a centre of anti-symmetry, the movement on one side of the centre is opposite to that on the other side. Thus half the cross-colour zone plates appear to be expanding and the other half, contracting.

If the size of the main zone plates is adjusted as in Fig. 7 then the chrominance carriers appear at ¼ and ¾ of the vertical distance from the horizontal axis to the top of the figure. The centres of the outer chrominance zone plates are then just visible.

In passing, it may be remarked that all the above arguments apply equally to the NTSC system with appropriate modifications. Thus the carrier positions of the two sorts of chrominance are coincident at a point half-way between the horizontal axis and the top of the figure; the movement is all at 15 Hz in the case of a 525/60 system.

Such a display can be used for a quick assessment of the effect of any cross-colour-reducing technique. It also shows how one type of PAL cross-colour can effectively mask another type (for example the coarse type dominates at the carrier positions). The display can also be used to show the effect on cross-colour of sub-Nyquist sampling systems[5] and systems which separate PAL into sub-Nyquist-sampled components.[6] Experience has shown that normal picture material is often slow to reveal increased sensitivity of systems to cross-colour. However, when it does occur it is often extremely objectionable. The zone plate must therefore be regarded, in this application, as a diagnostic tool giving 'advanced warning' of objectionable effects.

### 2.5. Standards conversion

When signals are converted from one standard to another, whether it be a change of element, line or field rate, various extraneous spectra are produced. These occur either because the interpolation process is inadequate or because the primary source already contains aliasing. The positions and strengths of these extraneous spectra are immediately observable on the zone plate.

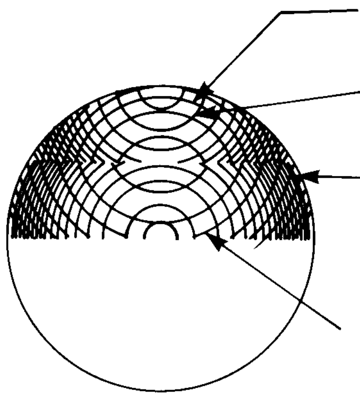


*Fig. 12 - The appearance of a zone plate converted in line standard from 625 to 405 lines when the interpolation is perfect on a field-by-field basis*

For example, in line-rate conversion from 625 to 405 lines where the interpolation is on a field-by-field basis, it can be shown that even if the interpolation is perfect for inputs below 156 c/ph, inputs beyond 156 c/ph produce steady alias components.[7] These are at a constant frequency difference of 110 c/ph (the difference of the field spatial frequencies) below the input. When the flickering alias components are taken into account the resultant 405-line display appears as in Fig. 12.

If the interpolation is imperfect then extra alias components appear, caused by the vestiges of the higher-order spectra in the original signal and their frequency translations. The strongest of these are the first-order spectra centred on ±312½ c/ph which, after translation, yield steady spectra centred on ±110 c/ph. Taking into account the inadequately attenuated input components lying beyond the interpolation cut-off frequency the extra components appear as in Fig. 13.

The positions of these components can be predicted quite simply by taking differences of multiples of the input and output sampling frequencies. Thus the causes of the most subjectively disturbing effects can be identified.

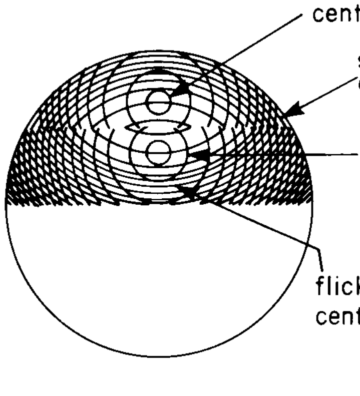


*Fig. 13 - The extra patterns which appear in Fig. 12 when the interpolation is imperfect*

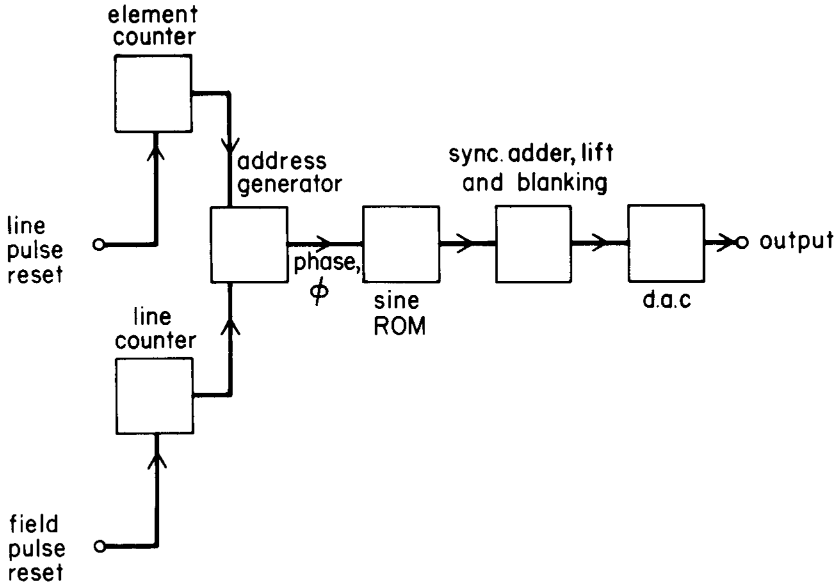


*Fig. 14 - The block diagram of an electronic zone plate pattern generator*

## 3. Electronic generation of the zone plate

### 3.1. Introduction

As mentioned earlier it is highly desirable, unless measuring source properties, to have an electronically-generated pattern. This is free of source imperfections and, if digital, has stable and precise characteristics. In particular it is possible to produce a sine-wave pattern of full amplitude over the whole frequency range of the transmission system.

The intensity variation of a sine-wave zone plate can be described by the expression

$$E(x,y) = \cos \phi$$

where $\phi$ is a function of the spatial co-ordinates of the picture. The function $\cos \phi$ can be stored in a read-only memory (ROM) so that the memory address is proportional to the phase, $\phi$. The problem of electronic generation is, thus, essentially, one of generating the memory address as a function of the spatial co-ordinates of the picture. Clearly, these are related to the element and line numbers of the scan and so the schematic circuit of a generator is as shown in Fig. 14. In the address generator, element and line counters, reset by line and field pulses respectively, are used to provide a measure of the horizontal and vertical position. If the sampling structure is orthogonal then the element counter output is simply proportional to the horizontal position. Non-orthogonal structures, however, require more complex arrangements involving the lowest common multiple of the sampling and line frequencies.

### 3.2. The circular zone plate

As shown in Appendix 1 the phase function for a circular zone plate is given by

$$\phi = \pi(x^2 + y^2)/r_0^2 \qquad (1)$$

This equation shows that the address can be formed by the addition of horizontal and vertical terms each proportional

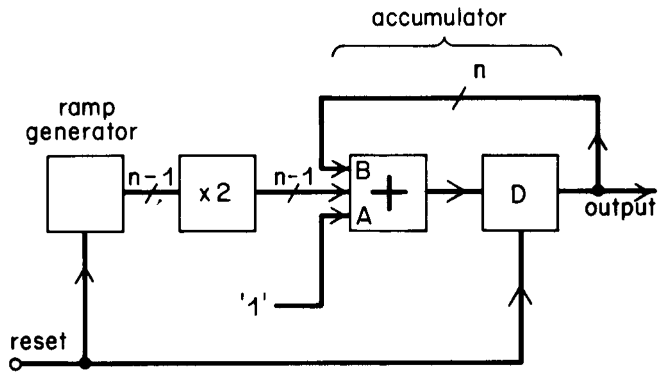


*Fig. 15 - A circuit for obtaining a sequence of squares*

to the square of the distance from the origin. Thus it is only necessary to generate the sequence of squares 1,4,9,. . . in response to the element and line counts. Fig. 15 shows a circuit for doing this based on the identity

$$(x + 1)^2 \equiv x^2 + 2x + 1$$

Because a finite number of bits is used in the accumulator the sequence so generated behaves in a modulo fashion which is just what is required of a phase measure. If $n$ bits are used the modulo base is, of course, $2^n$. What is not so obvious is that the sequence becomes palindromic after $2^{n-2}$ terms and repeats after a further $2^{n-2}$ terms. The reader may care to verify that for any modulo base, $m$, the squares of the natural numbers have this property except that the repeat unit is $m$ terms ($2^n$ is a special case). Looked at another way, this must be so because the sample pattern of a zone plate must repeat according to the observations of Section 2.3.

The address is formed by the addition of two such sequences, one each for the horizontal and vertical components. Circularity will not, however, be obtained unless the horizontal sampling frequency is equal, in absolute spatial terms, to the vertical, i.e. 14·74 MHz for system I and, in general, an elliptical pattern is produced. Likewise

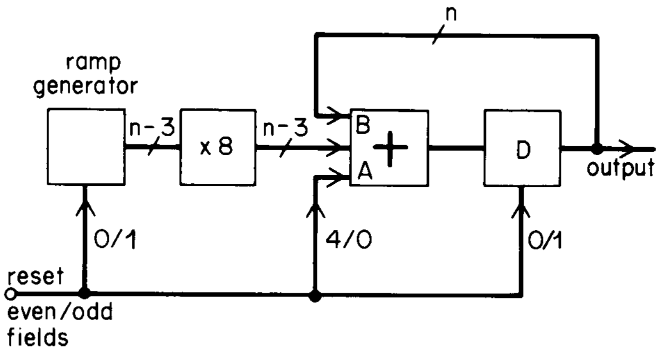


*Fig. 16 - A modification of the circuit of Fig. 15 to allow for interlaced scanning*

the horizontal and vertical repeat units will, in general, be unequal.

Interlaced scanning introduces a complication because the required vertical sequence consists of alternate terms of the horizontal. The circuit of Fig. 15 will serve, however, with a slight modification of the input as shown in Fig. 16. In either case the accumulator must be reset at the same time as the input count.

If the resetting is done at the beginning of the line and field this produces a pattern centred on the corner of the picture. Depending on the repeat units, however, other patterns may be seen. This is generally desirable because it ensures that the coarser spatial frequencies are displayed near the edge of the picture where the resolution is poorest and also any 'glitches' caused by the resetting are not displayed.

It can be seen that the generation of a circular zone plate is complicated by the need to add separate horizontal and vertical terms and especially by the need for a particular horizontal sampling frequency. This last disadvantage can only be surmounted by considerably more complex processing.

### 3.3. The hyperbolic zone plate*

As shown in Appendix 1 the phase function for a hyperbolic zone plate is given by

$$\phi = 2\pi xy/r_0^2 \qquad (2)$$

Compared with Equation (1) the phase function of Equation (2) is considerably easier to generate. As it is proportional to both $x$ and $y$ it can be obtained simply by accumulating $I$ on line one, $2I$ on line two and so on, where $I$ is an integer. The single constant of proportionality is determined by $I$, the number of accumulator bits, $n$, and the horizontal and vertical sampling intervals, $X$ and $Y$. The horizontal and vertical repeat units of the pattern, corresponding to the fundamental periods of the phase function are $2^nX/I$ and $2^nY/I$ as shown in Fig. 17 and

$$r_0^2 = 2^nXY/I$$

If $I$ equals unity the largest pattern magnification is obtained and the number of accumulator bits, $n$, is chosen to make this suitable. Note that if $X$ alone is changed (corresponding to a change in digital sampling frequency) the aspect ratio of the repeat cell and the spatial rate of change of frequency are both altered but, as shown in Appendix 1, the rate of change vertically is always equal to that horizontally. This means that this pattern, unlike the circular zone plate, preserves its equal horizontal and vertical scaling in spite of variations in the generating parameters.

Fig. 18 shows the effect of varying the parameter $I$ whilst using 12 accumulator bits and constant sampling frequencies of 851 samples/line and 625 lines/picture. This figure comprises photographs taken of a monochrome display with an exposure time of 2 field periods. The patterns centred on A, B, C, D, E and F are artifacts and are caused by the interaction of the picture detail with the printing screen. The patterns centred on 312½ c/ph in

* Proposed by M. Weston.

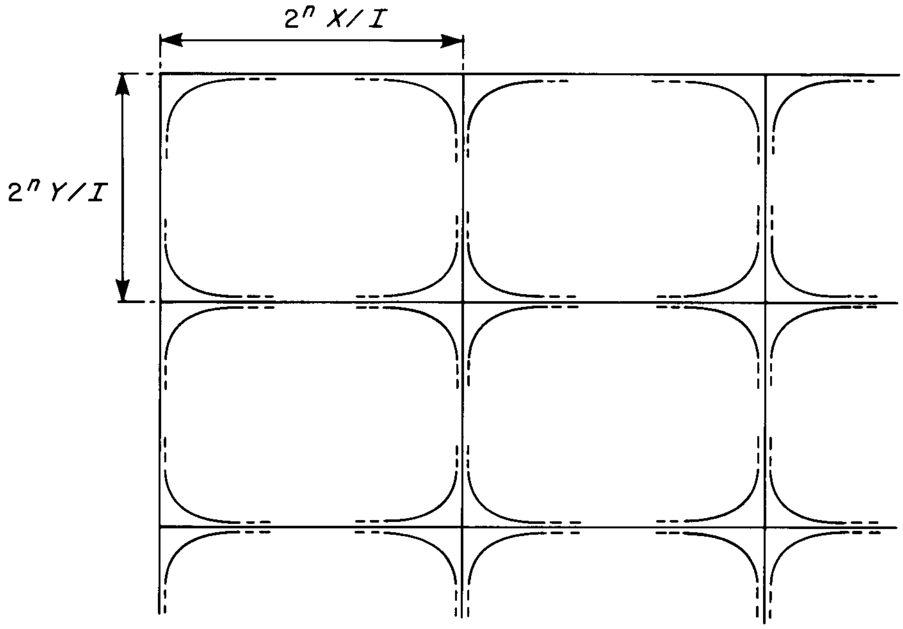


*Fig. 17 - The repeat units of the electronically generated zone plate pattern*

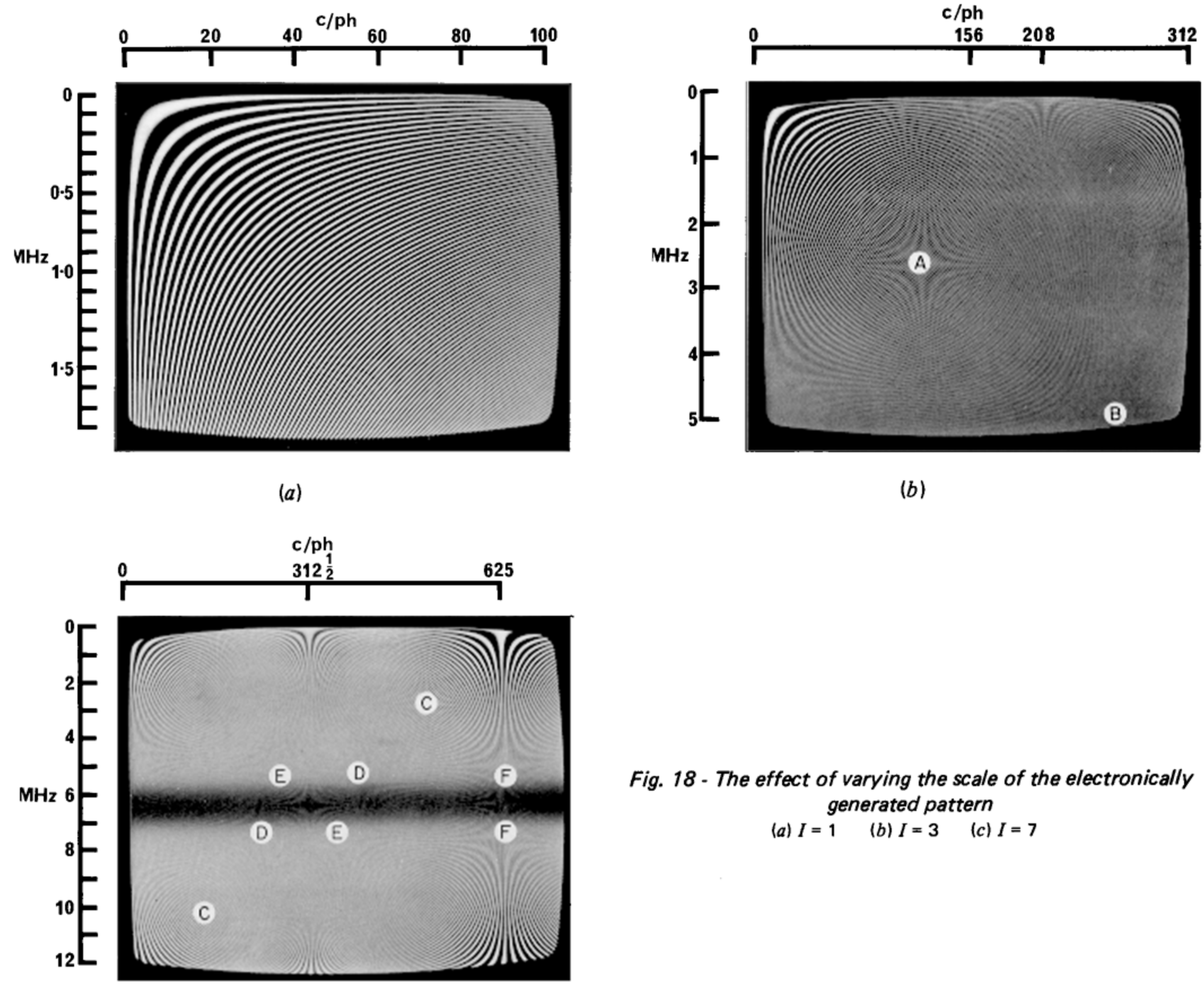


*Fig. 18 - The effect of varying the scale of the electronically generated pattern*
($a$) $I$ = 1 ($b$) $I$ = 3 ($c$) $I$ = 7

Figs. 18($b$) and ($c$) would not be recorded if the gamma of the display were unity because the integral of the light from the two fields, which is equivalent to a sampling operation of 625 c/ph, can represent frequencies up to 312½ c/ph. Because of the tube gamma, however, the displayed intensity contains harmonics which cannot be adequately represented and so the second harmonic causes a zero-frequency beat. This mechanism is also responsible for the patterns centred on 156 and 208 c/ph, caused by the 4th and 3rd harmonics respectively. The dark band in Fig. 18($c$) is caused by the action of the 5·5 MHz low-pass filter used in the generator. The circuit diagram of the hyperbolic zone plate generator used to produce these figures is given in Appendix 2.

## 4. Conclusions

This report has sought to explain the properties of the zone plate and has shown that it can quickly give an accurate picture of the subjective effect of any frequency characteristic arising from filtering or sampling. One form of zone plate is very easy to generate electronically and provides a stable, precise source for rapid quantitative and subjective measurement.

## 5. References

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## Appendix 1

### Derivation of the fundamental properties of circular and hyperbolic zone plate patterns

The zone plate derives its name from a phenomenon associated with the study of light as a wave motion. Fundamental to this study is the idea that a radiating spherical wavefront can be divided into annular zones each of which contributes to the optical disturbance at a point beyond the wavefront with alternating phase. If the radius of the wavefront and the distance from the wavefront to the point are both much greater than the wavelength of the light the wavefront can be considered plane over a substantial number of zones. Then it can be shown that the radius of the $n$th zone is given by

$$r_n = r_o\sqrt{n} \tag{3}$$

where $r_o$ is the radius of the first zone. If alternate zones are obscured the resulting pattern is the circular zone plate of Fig. 1.

The spatial wavelength of the pattern, $\lambda_n$, associated with the $n$th radius is given by

$$\begin{aligned}\lambda_n &= r_{n+1} - r_{n-1}\\ &= r_o(\sqrt{n+1} - \sqrt{n-1}) \text{ from (3)}\end{aligned}$$

If $n$ is large then $\lambda_n$ is given approximately by

$$\lambda_n = r_o/\sqrt{n}$$

at a distance $r_n$. Using (3) this may be written

$$\lambda_n = r_o^2/r_n$$

The spatial frequency, $f_n$, associated with the $n$th radius is given by

$$f_n = 1/\lambda_n = r_n/r_o^2$$

which is proportional to $r_n$. Neglecting the discrete nature of the pattern we may say that the magnitude of the spatial frequency vector at any point is proportional to the distance from the origin, and the direction of the spatial frequency vector is the same as that of the radius vector i.e.

$$\mathbf{f} = \mathbf{r}/r_o^2$$

Now the spatial frequency vector is related to phase by the equation

$$\mathbf{f} = (2\pi)^{-1}\nabla\phi$$

$$\text{or } \phi = 2\pi\int \mathbf{f}\cdot d\mathbf{r}$$

So for the circular zone plate the phase function is given by

$$\begin{aligned}\phi &= (2\pi/r_o^2)\int r\,dr\\ &= \pi r^2/r_o^2\\ &= \pi(x^2+y^2)/r_o^2\end{aligned} \tag{4}$$

The horizontal and vertical frequency components are given by

$$\begin{aligned}f_x &= (2\pi)^{-1}\partial\phi/\partial x \qquad & f_y &= (2\pi)^{-1}\partial\phi/\partial y\\ &= x/r_o^2 & &= y/r_o^2\end{aligned} \tag{5}$$

which shows that horizontal and vertical frequencies are proportional to horizontal and vertical distances respectively.

By extension of Equation (4) the phase function for an unskewed elliptical zone plate is given by

$$\phi = \pi(x^2/r_1^2 + y^2/r_2^2)$$

where $r_1$ and $r_2$ are the semi-axes.

By analogy the phase function for a hyperbolic zone plate is

$$\phi = \pi(x^2/r_1^2 - y^2/r_2^2)$$

where the asymptotes are given by

$$y = \pm(r_2/r_1)x$$

Putting $r_1 = r_2 = r_o$ gives the rectangular hyperbolic phase function

$$\phi = \pi(x^2 - y^2)/r_o^2$$

where the asymptotes bisect the quadrants.

If the pattern is rotated through 45° so that the asymptotes coincide with the $x$ and $y$ axes the phase function becomes

$$\phi = 2\pi xy/r_o^2 \tag{6}$$

The horizontal and vertical frequency components are given by

$$\begin{aligned}f_x &= (2\pi)^{-1}\partial\phi/\partial x \qquad & f_y &= (2\pi)^{-1}\partial\phi/\partial y\\ &= y/r_o^2 & &= x/r_o^2\end{aligned} \tag{7}$$

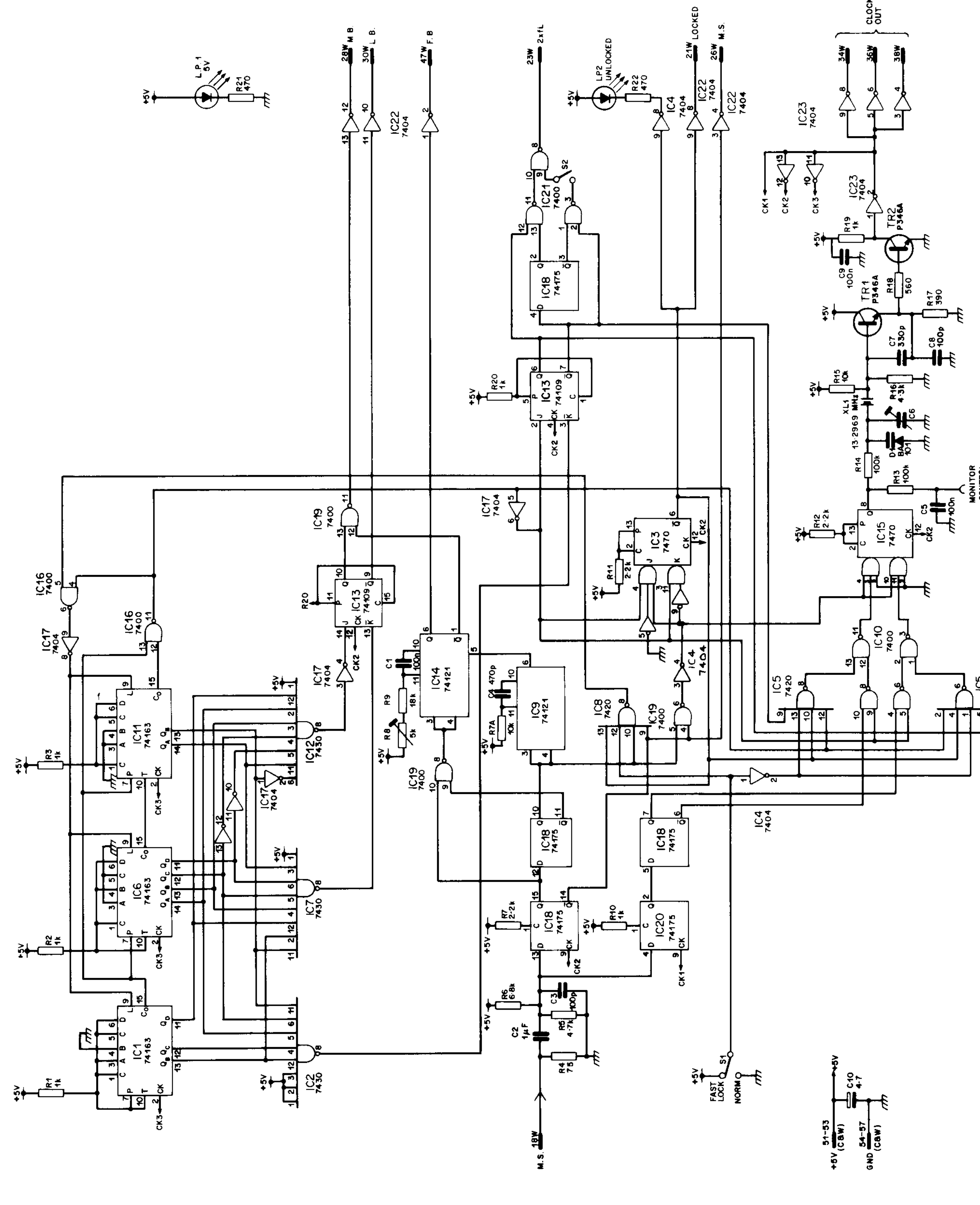
which shows that horizontal and vertical frequencies are proportional to vertical and horizontal distances respectively.

## Appendix 2

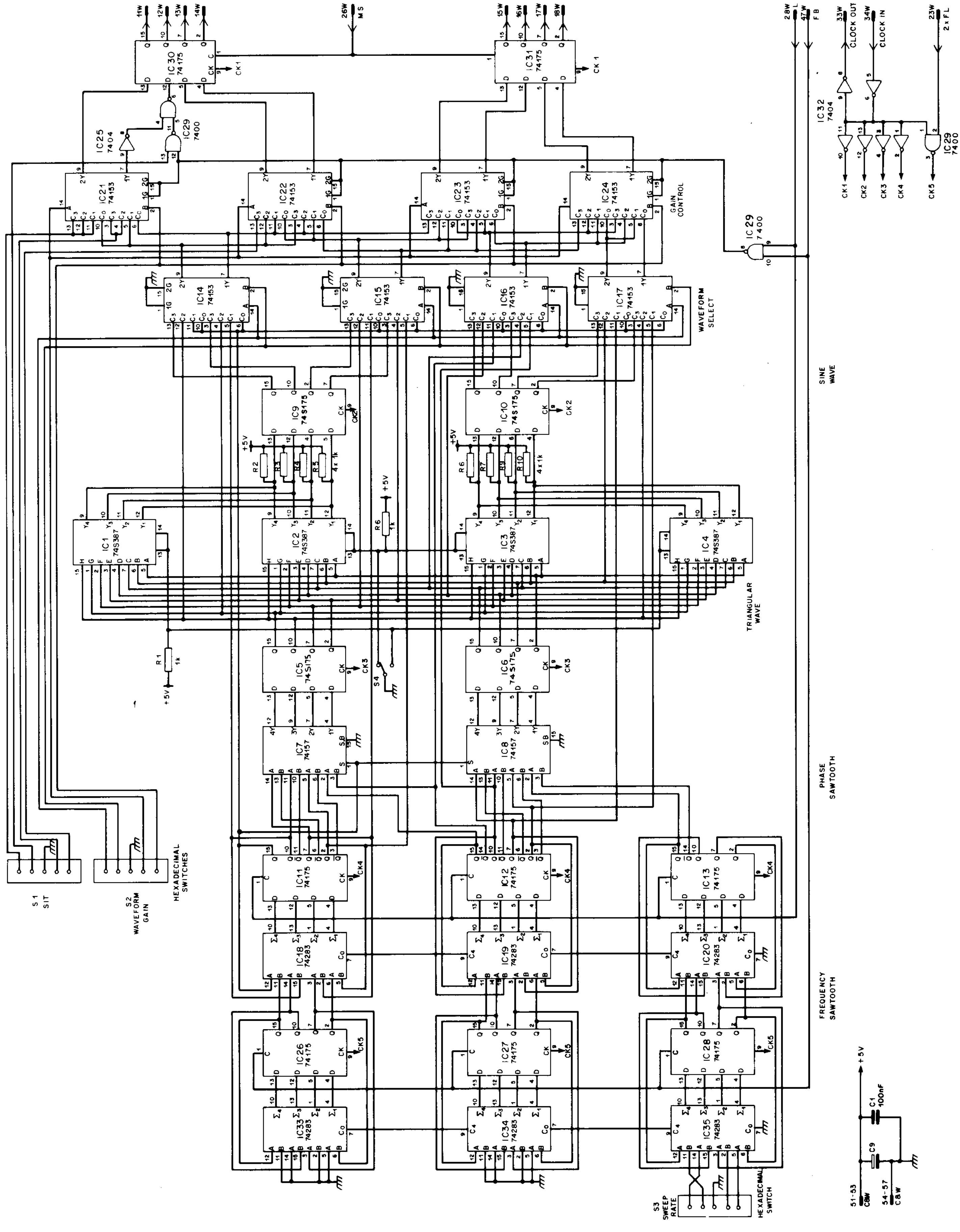
### Circuit diagrams of hyperbolic zone plate generator

There are two circuit diagrams. The first describes the circuit which takes a feed of mixed synchronising pulses and generates line-locked clock pulses at 851 times line frequency together with twice line frequency, mixed sync., line, field and mixed blanking waveforms at TTL levels. The second diagram describes the circuits which generate the actual zone plate pattern. The pattern waveform can be sinusoidal, square or triangular with an amplitude which can be set to one of several binary-related values.

SMW/VY



CLOCK OUT
MONITOR CONTROL SIGNAL
UNLOCKED
LOCKED
FAST LOCK
NORM



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